Using Macaulay2 effectively

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Macaulay2 project in a nutshell

History

- Written by Dan Grayson and myself, starting in the 1990's.
- Aim: Computational support for research in algebraic geometry, commutative algebra, and related fields (including applications)
- Now a community effort: more than 150 M2 packages written by users
- About 2 Macaulay2 workshops per year

Macaulay2 strong points

What can Macaulay2 do?

- Compute with ideals and matrices of polynomials
- Numerical information: dimension, degree, Hilbert function and polynomial
- Key engine: Groebner bases
- Applications of Groebner bases: Elimination of variables, Fiber of regular and rational maps, saturation, Hilbert series and polynomials, syzygies, free resolutions.
- Expressive user language (Dan will delve into more detail tomorrow)
- Packages: Greg will go into this in detail on Saturday. Users have now written about 125-150 such packages.

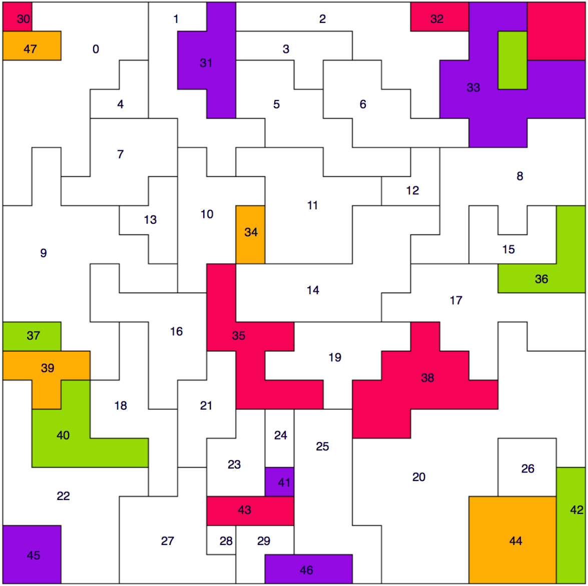
Some key packages

- Numerical algebraic geometry (several packages, see Anton's talk, Robert's talk)
- `Polyhedra`
- `NormalToricVarieties`
- Algebraic statistics (several packages, see Thomas' and Carlos' talks)
- Applications to Biology (see Elizabeth's talk)
- Simplicial complexes, graphs, posets, visualization, Schur polynomials
- Homological algebra, cohomology of sheaves in algebraic geometry (Core, BGG, TateOnProducts, BoijSoederberg)
- Weyl Algebra and D-modules
- ...

Do `viewHelp` inside Macaulay2 to see the entire list of packages distributed with Macaulay2.

Example to get started. Working with ideals

Puzzlemaniak, a game on my ipad: coloring maps



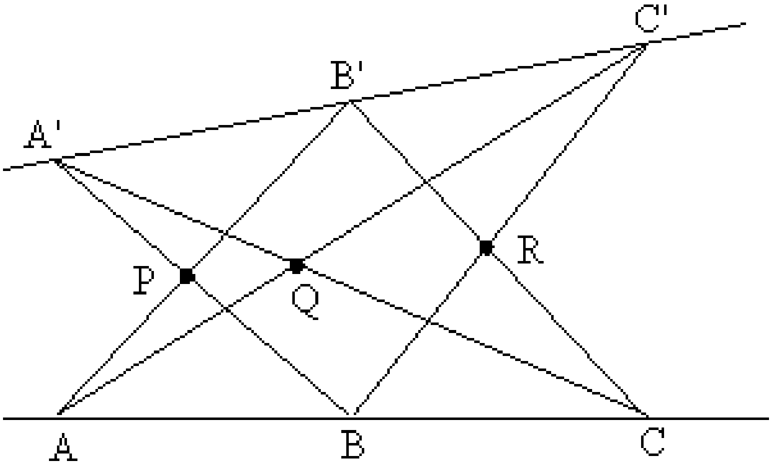
Our goal

Let's translate this problem into algebra, and use Gröbner bases to solve it.

Algebraic translation

- Give each color a number, e.g: Red = -1, Orange = 0, Green = 1, and Purple = 2.
- Define a variable for each of the 48 regions. Let $R = \mathbb{Q}[x_0, \ldots, x_{47}]$.
- Construct an ideal whose solutions are precisely all of the allowed 4-colorings.
- For each region x, add in a polynomial $f(x) = (x+1)x(x-1)(x-2)$.
- For each adjacent pair of regions x and y, add in a polynomial $g(x, y)$ which states that the two regions have different colors.
- For each region already colored, add in a polynomial of the form $x - a$.
- These polynomials form an ideal $I \subset R$. Compute a Gröbner basis and find the solutions.

Example: Pappus' theorem



Example: Investigating ideals with high regularity