Applied M2 Tutorials - Degree Exercises

July 27, 2017

1. Using `Macaulay2`, compute the degree of the variety in $\mathbb{C}^4$ parametrized by

$$\phi : \mathbb{C}^2 \to \mathbb{C}^4$$

$$(s,t) \mapsto (s,t,s^2,st)$$

first using a symbolic method and then a numerical method.

2. Let X be an affine variety in $\mathbb{C}^n$ with dimension d. Let a be the lead coefficient of the Hilbert polynomial of $K[x_1, \dots, x_n]/\mathbf{I}(X)$. Prove

$$d!a = \#(X \cap L)$$

where L is a generic affine linear space with $\dim L = n - d$.

3. Considering $\mathrm{O}(n)$ as the affine variety cut out by the relation $M^T M = \mathrm{Id}$, what is the Bézout bound on the degree of $\mathrm{O}(n)$? What is the mixed-volume bound?

4. The Grassmannian $\mathbf{Gr}(k, \mathbb{C}^n)$ can be considered as an affine variety in $\mathbb{C}^{\binom{n}{k}}$ under the Plücker embedding. Compute the degree of $\mathbf{Gr}(k, \mathbb{C}^n)$ for whatever values of n and k you can.

5. The group of symplectic matrices $\mathrm{Sp}(n)$ can be considered as an affine variety in the space of all real $2n \times 2n$ matrices. Compute the degree of $\mathrm{Sp}(n)$ for whatever values of n you can. Find a formula in n for the degree of $\mathrm{Sp}(n)$.