Fitness: Phylogenetic Trees and Reaction Networks

Phylogenetic Trees

1. Compute the toric phylogenetic ideal for the CFN model on the 7-leaf tree pictured in Figure 1.

2. Compute the phylogenetic toric ideal for the Jukes-Cantor model on the 5-leaf claw tree. It is known that the toric ideal for the Jukes-Cantor model on the n-leaf claw tree is generated by quadrics. Verify this theorem for $n = 5$ and $n = 6$.

3. Find the secant ideal corresponding to the Jukes-Cantor two-tree mixture model where both constituent trees are the 4-leaf tree in Figure 2.

Reaction Networks

1. Find the steady-state degree of the chemical reaction network pictured in Figure 3.

2. Let $\mathcal{N}$ be a reaction network with n species. Fix the rate constants, and let $\mathcal{V}_{\mathcal{N}} \subseteq \mathbb{C}^n$ be the variety defined by the steady-state equations of $\mathcal{N}$. Let $u = (u_1, \ldots, u_n)$ be a data point. In applications, we would like to minimize the weighted Euclidean distance between the observed data u and $\mathcal{V}_{\mathcal{N}}$, i.e. for $\lambda_i \in \mathbb{R}$, we would like to solve the following constrained optimization problem

$$\text{Minimize} \sum_{i=1}^{n} \lambda_i (u_i - x_i)^2 \text{ subject to } x \in \mathcal{V}_{\mathcal{N}}. \tag{1}$$

   The *Euclidean distance degree* is the the number of critical points of (1) for generic λ and u.

   Load the reaction network $\mathcal{N} =$`oneSiteModificationB()` and fix the rate constants. Determine the Euclidean distance degree of $\mathcal{N}$.

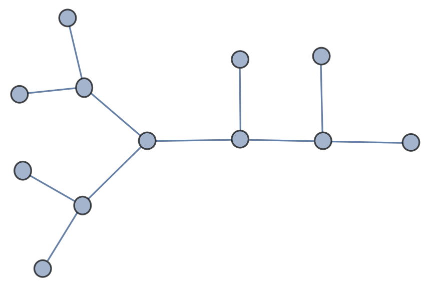

Figure 1: 7 leaf tree

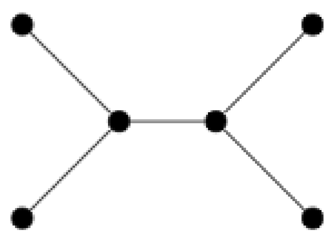

Figure 2: Quartet

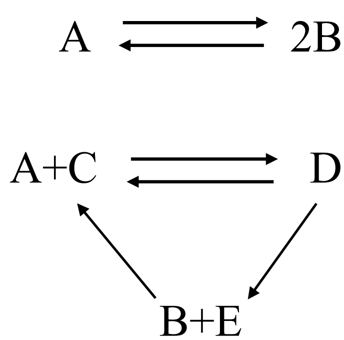

Figure 3: Chemical reaction network